

Roots of Polynomials

If $ax^2 + bx + c = 0$ has roots α, β

$$a(x - \alpha)(x - \beta) = 0$$

$$a(x^2 - (\alpha + \beta)x + \alpha\beta) = 0$$

$$ax^2 - a(\alpha + \beta)x + a\alpha\beta$$

$$a = a \quad \therefore \alpha + \beta = \frac{-b}{a}$$

$$b = -a(\alpha + \beta)$$

$$c = a\alpha\beta$$

$$\alpha\beta = \frac{c}{a}$$

If $ax^3 + bx^2 + cx + d = 0$ has roots α, β, γ

$$a(x - \alpha)(x - \beta)(x - \gamma) = 0$$

$$a(x^3 - x^2(\alpha + \beta + \gamma) + x(\alpha\beta + \alpha\gamma + \beta\gamma) - \alpha\beta\gamma) = 0$$

$$a = a$$

$$\therefore \frac{-b}{a} = (\alpha + \beta + \gamma)$$

$$b = -a(\alpha + \beta + \gamma)$$

$$c = a(\alpha\beta + \alpha\gamma + \beta\gamma)$$

$$\frac{c}{a} = \alpha\beta + \alpha\gamma + \beta\gamma$$

$$d = -a\alpha\beta\gamma$$

$$\frac{-d}{a} = \alpha\beta\gamma$$

If $ax^4 + bx^3 + cx^2 + dx + e = 0$ has roots $\alpha, \beta, \gamma, \delta$,

$$\alpha + \beta + \gamma + \delta = -\frac{b}{a}$$

$$\alpha\beta + \alpha\gamma + \alpha\delta + \beta\gamma + \beta\delta + \gamma\delta = \frac{c}{a}$$

$$\alpha\beta\gamma + \alpha\beta\delta + \alpha\gamma\delta + \beta\gamma\delta = -\frac{d}{a}$$

$$\alpha\beta\gamma\delta = \frac{e}{a}$$